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TACHYON CONDENSATES AND ANISOTROPIC UNIVERSE

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ABSTRACT

We investigate the cosmological solutions in closed bosonic string theory in the presence of non zero tachyon condensate. We specifically obtain time dependent solutions which describes an anisotropic universe. We also discuss the nature of such time dependent solutions when small tachyon fluctuations around the condensate are taken into account.

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String theory is likely to provide us with a consistent quantum theory of gravity. The issue of understanding the structure of space-time singularities is one of the important questions and one hopes to have a better understanding of the nature of such singularities by studying them in the context of string theory. As we know, the lowest lying fields in the case of closed bosonic string theory are the metric, antisymmetric tensor field, dilaton and a tachyon. In the sigma model approach, the requirement of vanishing beta function equations for various background fields ensures conformal invariance of the theory^[1]. On the other hand, these beta function equations can be derived as the target space equations of motion for the tachyon T , dilaton Φ , metric $G_{\mu\nu}$ and the anti symmetric tensor field $B_{\mu\nu}$ from the low energy effective action. One then looks for consistent solutions of the beta function equations which represent nontrivial backgrounds for string propagation. These solutions in string theory are qualitatively quite different from their corresponding counterpart in general theory of relativity due to the presence of a nontrivial dilaton field. String propagation in cosmological backgrounds have also been discussed before[§].

The beta function analysis including the tachyon has been considered in ref^[8]. The existence of the tachyon destabilizes the canonical 26-dimensional vacuum. But it is quite possible that the collective effects may stabilize the closed string in another ground state containing a non zero tachyon expectation value. In^[9], such a possibility was investigated starting from a covariant string field theory through mass-level truncation scheme. It was shown that upto cubic order in tachyon field,

§ see for example^{[2] [3] [4] [5] [6] [7]}.

the string field theory gets a contribution to the tachyon potential term of the form

$$V(\hat{T}) = -\frac{2}{\alpha'}\hat{T}^2 + \frac{\hat{g}}{6}\hat{T}^3. \quad (1)$$

Here \hat{T} is the tachyon field appearing in the string field theory and \hat{g} is the three point tachyon coupling at zero momentum. From (1), it is clear that the potential has a minimum at nonzero value of tachyon along with a maximum at $\hat{T} = 0$. It is possible that higher order corrections to (1) will destroy the structure of the potential, but nevertheless, it is very tempting to assume that such a non-trivial minimum exists even when the higher order corrections in the tachyon potential are taken in to account.

When the tachyon is settled at the minimum of the potential, the nature of an isotropic universe has been discussed recently in^[10]. Consequences on the metric for small tachyon fluctuations and possible cosmological scenario has also been considered^[11]. In this note, we find that such a tachyon condensate does not isolate an isotropic universe. Infact, there are solutions of string equations which describes anisotropic metric leading to an anisotropic evolution of the universe. Such possibility exists even when the tachyon slowly fluctuates around its non zero vacuum expectation value.

We start with Bianchi-I type of anisotropic metric with the line element given by

$$ds^2 = -dt^2 + \sum_i a_i^2(t) dx_i^2 \quad (2)$$

where, $i = 1, \dots, d-1$, $a_i(t)$'s are the scale factors in the presence of non-zero tachyon condensate.

The motivation to consider the anisotropic cosmological model is due to lack of adequate explanation for the high degree of isotropy in the universe as observed by experiments. There are arguments given previously that adiabatic cooling and viscous dissipation might be the reason for destroying the anisotropy of the universe. Also the process of quantum particle creation in the anisotropic expansion might have caused a stronger damping of the anisotropy in the early universe. Though the Friedman- Robertson-Walker cosmological model is satisfying enough from the point of view of experiment, however it does not give a satisfactory answer that why is the universe so homogeneous and isotropic on large scales. So one needs to go beyond the isotropic case to understand this phenomena. Our aim is to see if such an anisotropic model is allowed by string theory. Given the ansatz for the metric, we look for solutions of the beta function equations for $G_{\mu\nu}$, Φ and T . We assume that the dilaton and tachyon fields are functions of time only and we consider the d dimensional space time for our analysis.

The action for a string propagating in a background of graviton, dilaton and tachyon condensates is given by,

$$S = -\frac{1}{2\pi\alpha'} \int d^2\sigma \sqrt{g} [g^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} + \frac{1}{2} \alpha' T - \frac{1}{4} \alpha' R^{(2)} \Phi] \quad (3)$$

Here g_{ab} is the two dimensional metric in the world sheet and $R^{(2)}$ is the world sheet curvature scalar. The volume two form on the world sheet has been neglected here, but in principle we could have chosen a nonzero $B_{\mu\nu}$ field. The background fields are the metric $G_{\mu\nu}$, dilaton Φ and the tachyon T and they basically act like couplings in the corresponding non linear sigma model. The one loop beta function equations for the metric, tachyon and dilaton are respectively given by,

$$\begin{aligned}
R_{\mu\nu} &= \nabla_\mu \nabla_\nu \Phi + \nabla_\mu T \nabla_\nu T; \\
\nabla^2 T + \nabla_\mu \Phi \nabla^\mu T &= V'(T); \\
\hat{c} &= R - (\nabla \Phi)^2 - 2\nabla^2 \Phi - (\nabla T)^2 - 2V(T).
\end{aligned} \tag{4}$$

where, $\hat{c} = \frac{2(d-26)}{3\alpha'}$, $V'(T) = \frac{\partial V(T)}{\partial T}$ and $V(T)$ is the tachyon potential. As discussed before, these background field equations can be derived from the target space effective action in d dimensions, which is given by,

$$S_{eff} = -\frac{1}{2\kappa^2} \int d^d x \sqrt{G} e^\Phi [\hat{c} - R - (\nabla \Phi)^2 + (\nabla T)^2 + 2V(T)] \tag{5}$$

where, $\kappa^2 = 8\pi G_N$ when $d = 4$. This effective action has been written in the string frame, where $e^{\frac{\Phi}{2}}$ serves as the string coupling constant. This action can be represented in the so called Einstein frame through the following transformation,

$$G_{\mu\nu}^E = e^{-\frac{2\Phi}{d-2}} G_{\mu\nu}^\sigma \tag{6}$$

Having given the frame work, let us look at the various beta function equations corresponding to the ansatz for the metric describing the anisotropic universe. The equations for the tt and ii components of the metric are respectively given by,

$$\begin{aligned}
\left(\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_{d-1}}{a_{d-1}}\right) + \ddot{\Phi}(t) - (\dot{T})^2 &= 0; \\
a_i \ddot{a}_i + a_i \dot{a}_i \sum_{j \neq i} \left(\frac{\dot{a}_j}{a_j}\right) + a_i \dot{a}_i \dot{\Phi} &= 0
\end{aligned} \tag{7}$$

The ti component of the metric equation vanishes identically. Tachyon equation is given by,

$$\ddot{T} + \dot{T}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \dots + \frac{\dot{a}_{d-1}}{a_{d-1}}\right) + \dot{\Phi}\dot{T} + V'(T) = 0 \quad (8)$$

where prime denotes differentiation w.r.t. T and dot denotes derivatives w.r.t. time. Finally the dilaton equation is given by,

$$\hat{c} = (\dot{\Phi})^2 + \ddot{\Phi} + \dot{\Phi}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \dots + \frac{\dot{a}_{d-1}}{a_{d-1}}\right) - 2V(T) \quad (9)$$

As we discussed before, at the extremum of the tachyon potential ($V'(T) = 0$), $T = T_0$ (constant) can also be a consistent solution apart from the case of $T = 0$. We note that, for $T = T_0$, $V(T_0) \neq 0$ and $V''(T_0)$ is positive which ensures a stable solution. We define the Hubble parameter H_i as, $H_i = \frac{\dot{a}_i}{a_i}$ and h_i 's are defined as $h_i(t) = \ln(\sqrt{\beta} \frac{\dot{a}_i}{a_i})$, where β is a positive number. Eliminating Φ from equation eqn.(7), we find that they can be written in a more compact form (in terms of h_i 's) namely,

$$\beta \ddot{h}_i = \sum_{i=1}^{d-1} e^{2h_i} \quad (10)$$

This equation can be integrated to give,

$$(\dot{h}_i)^2 = \frac{1}{\beta} \sum_{i=1}^{d-1} e^{2h_i} + k \quad (11)$$

where, k is an integration constant and $\dot{h}_1 = \dot{h}_2 = \dots \dot{h}_{d-1}$ (assuming that one finds a consistent solution for $a_i(t)$'s, which satisfy the above equality). In fact we

find that this is true. The dilaton can be written in terms of h_i 's as

$$\dot{\Phi} = -\dot{h}_i - \frac{1}{\sqrt{\beta}} \sum_{i=1}^{d-1} e^{h_i} \quad (12)$$

We now look for solutions of (11) for various values of k , namely $k = 0, k > 0$ and $k < 0$. The simplest case is for $k = 0$. We find that the consistent solutions for the scale factors and dilaton are given by,

$$a_1(t) = a_1^0 t^{p_1}; \quad a_2(t) = a_2^0 t^{p_2}; \quad \dots, a_{d-1}(t) = a_{d-1}^0 t^{p_{d-1}} \quad (13)$$

with the constraint,

$$\sum_{i=1}^{d-1} (p_i)^2 = 1 \quad (14)$$

The dilaton is given by,

$$\Phi(t) = [1 - (p_1 + p_2 + \dots + p_{d-1})] \log\left(\frac{t}{\sqrt{\beta}}\right) + \Phi_0 \quad (15)$$

where, a_1^0, a_2^0, \dots and Φ_0 are constants, p_i 's are real numbers. Substituting this into dilaton equation, we find that $\hat{c} + 2V(T_0) = 0$. Note that, the above choice of a_i 's leads to the most general solution and the condition $\dot{h}_1 = \dot{h}_2 = \dots = \dot{h}_{d-1} = \dot{h}$ is also satisfied as it should, in order to have a consistent solution of (12). The tachyon beta function equation is also satisfied for the choice $T = T_0 \neq 0$.

Next, we consider the case of positive k ($k > 0$). The solutions for a_i 's are given by,

$$\begin{aligned}
a_1(t) &= a_1^0 \left[\tanh\left(\frac{\sqrt{k}}{2}t\right) \right]^{p_1} ; & a_2(t) &= a_2^0 \left[\tanh\left(\frac{\sqrt{k}}{2}t\right) \right]^{p_2} ; \\
&\dots\dots a_{d-1}(t) &= a_{d-1}^0 \left[\tanh\left(\frac{\sqrt{k}}{2}t\right) \right]^{p_{d-1}}
\end{aligned} \tag{16}$$

Then for this choice of $a_i(t)$'s, the dilaton is found to be,

$$\begin{aligned}
\Phi(t) &= \Phi_0 + \left[1 + \sum_{i=1}^{d-1} p_i \right] \text{logcosh}\left(\frac{\sqrt{k}}{2}t\right) \\
&+ \left[1 - \sum_{i=1}^{d-1} p_i \right] \text{logsinh}\left(\frac{\sqrt{k}}{2}t\right)
\end{aligned} \tag{17}$$

As before, p_i 's satisfy the condition, $\sum_{i=1}^{d-1} (p_i)^2 = 1$. Substituting the above solutions in the dilaton beta function equation, we find,

$$\hat{c} + 2V(T_0) = k \tag{18}$$

Next, consider the case when k is negative ($k < 0$). In this case, the solutions for the scale factors are given by,

$$\begin{aligned}
a_1(t) &= a_1^0 \left[\tan\left(\frac{\sqrt{k}}{2}t\right) \right]^{p_1} ; & a_2(t) &= a_2^0 \left[\tan\left(\frac{\sqrt{k}}{2}t\right) \right]^{p_2} ; \\
&\dots\dots a_{d-1}(t) &= a_{d-1}^0 \left[\tan\left(\frac{\sqrt{k}}{2}t\right) \right]^{p_{d-1}}
\end{aligned} \tag{19}$$

The solution for dilaton is given by,

$$\begin{aligned}\Phi = \Phi_0 + [1 - \sum_{i=1}^{d-1} p_i] \log \cos(\frac{\sqrt{k}}{2}t) \\ + [1 - \sum_{i=1}^{d-1} p_i] \log \sin(\frac{\sqrt{k}}{2}t)\end{aligned}\tag{20}$$

with the condition, $\sum_{i=1}^{d-1} (p_i)^2 = 1$. The dilaton beta function equation in this case implies that, $\hat{c} + 2V(T_0) = -k$.

We notice that for small t , the behaviour of the solutions for the $k > 0$ and $k < 0$ cases are the same as that of $k = 0$ case. As t increases, the scale factors tend towards constant values namely a_1^0, a_2^0, \dots etc. (implying asymptotically flat space) and dilaton grows linearly with time. For $k < 0$, we find that the scale factors diverge for $t \rightarrow \frac{\pi}{\sqrt{k}}$, which implies that the universe expands to infinite size in finite amount of time. When all the p_i are equal, our solution reduces to that of the isotropic model considered by Kosteletzky and Perry^[10].

We also notice that the Kasner model for the anisotropic universe^[12] is a solution of the string beta function equations with a constant dilaton background. Kasner model in four dimensions is described by the metric,

$$ds^2 = -dt^2 + t^{2p_1} dx^2 + t^{2p_2} dy^2 + t^{2p_3} dz^2\tag{21}$$

In this metric, the p_i 's are real numbers, which satisfy the relation,

$$\sum_{i=1}^3 p_i = 1; \quad \sum_{i=1}^3 (p_i)^2 = 1\tag{22}$$

Note that, our most general solution also required the constraint $\sum_{i=1}^3 (p_i)^2 = 1$ in four dimensions. The constraint $\sum_{i=1}^3 p_i = 1$, automatically implies a constant

dilaton in our case. Here one of the p_i 's has to be negative to satisfy the constraint $\sum_i p_i = 1$ in the Kasner model. The limits are found to be,

$$-\frac{1}{3} \leq p_1 \leq 0; \quad 0 \leq p_2 \leq \frac{2}{3}; \quad \frac{2}{3} \leq p_3 \leq 1. \quad (23)$$

More generally, in the mixmaster universe (which was originally introduced to explain the inhomogeneous behaviour of the universe near a singularity)^[13], the Kasner exponents p_i 's can become functions of time near the singularity. The Kasner like behaviour exists for the exponents corresponding to some fixed u parameter. They are given by,

$$p_1(u) = -\frac{u}{1+u+u^2}; \quad p_2(u) = \frac{1+u}{1+u+u^2}; \quad p_3(u) = \frac{u(1+u)}{1+u+u^2} \quad (24)$$

with the constraint, $\sum_{i=1}^3 (p_i)^2 = 1$; $\sum_{i=1}^3 p_i = 1$. In the string theory frame work, the mixmaster metric is also found to be a consistent solution with a constant dilaton background. Remember that, we are doing all these analysis in a background where tachyon has a constant expectation value.

Finally, we consider small tachyon fluctuation around its background value and investigate the possibility of a viable inflationary scenario in the case of anisotropic universe. The tachyon potential given in (1) appears in non-polynomial string field theory. It is known that various fields that appear in this field theory and the corresponding sigma model coupling constants (or the fields in the low energy effective field theory derived from it) are related through complicated field redefinitions. But as far as the tachyon field in two theories are concerned, they are

related simply by^[10]

$$T = \frac{\hat{g}}{4}\alpha'\hat{T}, \quad V(T) = \frac{\hat{g}^2}{16}\alpha'^2\hat{V}(\hat{T})$$

so that,

$$V(T) = -\frac{2}{\alpha'}T^2 + \frac{2}{3\alpha'}T^3 \quad (25)$$

The inflationary scenario^[14] involving such a tachyon effective potential has been discussed recently in ref.^[11]. They ofcourse assumed a $k = 0$ Friedman-Robertson-Walker universe to start with. Here we will consider the anisotropic model as before. The tachyon plays the role of the scalar field in inflationary model, which undergoes a slow roll during the inflation. Since here we consider fluctuations of the tachyon around the condensate, we expect the solutions of the beta function equations to change, though we can not solve these equations exactly as we could do before. We rewrite the beta function equations in terms of h_i 's. Eliminating Φ from the beta function equations, we get,

$$(\dot{T})^2 = \ddot{h}_i - \frac{1}{\beta} \sum_{i=1}^{d-1} e^{2h_i}, \quad (26)$$

$$\ddot{T} - \dot{h}\dot{T} + V'(T) = 0 \quad (27)$$

Note that the l.h.s. of eqn.(26) was zero in the constant tachyon background case and equation (27) was identically zero as $T = T_0$ was the true minimum. We have written eqn. (26) assuming again that one finds the consistent solutions h_i 's, where all the h_i 's differ by constants only, which we have noticed in the previous case. We assume that as time t increases, the tachyon rolls towards the true minimum at $T = T_0$. This means that $(\dot{T})^2$ and \ddot{T} terms are small in the above consideration. We take T as a linear function of t , approximated by,

$$T = Ct + T_0 \quad (28)$$

where, C and T_0 are constants. \ddot{T} term then drops out from equation (27) given above and we have the following two equations to solve,

$$C\dot{h}_i - V'(T) = 0 \quad (29)$$

$$(\dot{h}_i)^2 - \frac{1}{\beta} \sum_{i=1}^{d-1} e^{2h_i} - 2C^2 h_i(t) = k' \quad (30)$$

where, k' is an integration constant. We have used the fact that all the \dot{h}_i 's are equal. It is difficult to find exact solutions of these equations. So we retain only upto order h_i terms in the exponential as has been done in ref.^[11]. Then we have the following equation to solve,

$$(\dot{h}_i)^2 - \frac{1}{\beta} [(d-1) + 2 \sum_{i=1}^{d-1} h_i] - 2C^2 h_i = k' \quad (31)$$

Since all the h_i 's differ by constants only, we can write eqn. (31) as,

$$(\dot{h})^2 - \frac{(d-1)}{\beta} [1 + 2h(t)] - 2C^2 h(t) = k' + k'' \quad (32)$$

where, k'' is the corresponding constant factor which we generate by writing $h_i(t) = h(t) + k_i$. $h(t)$ is nothing but the t dependent part of $\log(\sqrt{\beta} \frac{\dot{a}_i}{a_i})$, which we take to be equal for all the a_i 's. We can rewrite (32) as,

$$(\dot{h})^2 - C_1 h(t) - C_2 = 0 \quad (33)$$

where,

$$C_1 = \frac{2(d-1)}{\beta} + 2C^2; \quad C_2 = \frac{d-1}{\beta} + k' + k'' \quad (34)$$

Integrating (33), we obtain,

$$h(t) = h_0 \pm t\sqrt{C_1 h_0 + C_2} + \frac{1}{4}C_1 t^2 \quad (35)$$

And the solutions for h_i 's are given by,

$$h_i(t) = (h_0 + k_i) \pm t\sqrt{C_1 h_0 + C_2} + \frac{1}{4}C_1 t^2 \quad (36)$$

Even though h_i s differ from each other only by a constant, the corresponding a_i s can differ from each other in a time dependent manner. As a result it would generically lead to anisotropic universe.

In this letter, we have found consistent solutions of beta function equations which describes an anisotropic universe in the presence of nonzero tachyon condensate. We have also investigated the possibility of obtaining such solutions for the metric and dilaton for small tachyon fluctuations around its background value. This looks like a viable candidate for studying the inflationary cosmology in string theory context. Recently, time dependent solutions in string theory have been considered in ref.^[15], where the modulus field changes with time, thereby inducing a dynamical topology change in the theory. The time dependent radii there are analogous to the scale factors $a_i(t)$. Our solutions for the scale factors for the cases

$k = 0$, $k < 0$, $k > 0$ are similar to those in^[15]. One has to investigate in detail the tachyon induced inflation in an isotropic universe. It would also be interesting to find out an exact solution in the above anisotropic case, having a conformal field theory description of the underlying sigma model action.

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